Bar-Gain Boxes:
An Informative Illustration of the Pairing Problem

Kevin Burns

The MITRE Corporation, 202 Burlington Road, Bedford, MA 01730-1420, USA
kburns@mitre.org

Abstract. A practical problem in command and control is to assign assets (e.g., bomber planes) to targets (e.g., hostile sites), one-on-one, in order to optimize an overall operation. The asset-target pairing must be completed quickly (before targets act), and the expected effectiveness of an asset against a target depends on a number of factual and judgmental factors. Here I present a diagram called "Bar-Gain Boxes" designed to help people solve the problem. The diagram uses a matrix of boxes to illustrate the possible pairings, along with color-coded bars (in each box) to illustrate the gain associated with each individual asset-target pair. The diagram is informative because it displays algorithmic results and underlying reasons, for normative and alternative solutions.

1 Pairing Problem

A current challenge in command and control (e.g., Gulf Wars) involves assigning assets (e.g., attack aircraft) to time-critical targets (e.g., enemy entrenchments), typically one asset to each target. Given N assets and N targets, there are N! possible solutions, and with time-critical targets an effective (optimal) solution must be found quickly (before targets act).

The optimal solution can be computed by numerical methods, such as exhaustive enumeration (for problems with small N) or an auction algorithm [1], once a value function and its input data are specified. The value function gives the expected utility (gain) of an individual asset against an individual target. The problem is that, in practical applications, the value function is often incomplete and its input data are often uncertain. Thus, the final decision (on a set of N asset-target pairs) is made by a human being with help from a support system.

A typical support system provides a table display that lists the N targets along with the one asset (for each target) and associated value assigned by the normative (optimal) solution. The human's dilemma is that the normative solution is only as good as the system's value function, yet the system does not show how the value function affects either the normative solution or alternative solutions (which may be more optimal to the human, whose expertise may not be captured by the system's value function). Here I present a diagram that helps by displaying algorithmic results and underlying reasons, for normative and alternative solutions.

2 Bar-Gain Boxes

In a typical pairing problem, the value function that gives the gain (G) of pairing an asset to a target has three terms: $G = (U_T*P_T) - (U_O*P_O) - (U_A*P_A)$. U_T is the utility of an emergent, time-critical target (T) to which an asset (A) can be paired. P_T is the probability that A will be effective against T. Thus, U_T*P_T is the expected utility (score) of A against T. Similarly, U_O*P_O is the expected utility (cost) of diverting A from its original, scheduled mission. Finally, U_A*P_A is the expected utility (risk) of losing A, from threats by or near T.

In words, the value function can be expressed as follows: Pairing Gain (G) = Target Score (U_T*P_T) - Divert Cost (U_O*P_O) - Asset Risk (U_A*P_A). Using the standard military convention of "red" to denote enemy (Target Score) and "blue" to denote friendly (Asset Risk), and using "black" to denote the Pairing Gain and "yellow" to denote the Divert Cost, the value function can be expressed in colors as follows:

black (Pairing Gain) = red (Target Score) - yellow (Divert Cost) - blue (Asset Risk)

"Bar-Gain Boxes" (Fig. 1) is a color-coded diagram that illustrates the structure of the value function and pairing solutions. Fig. 1a shows the legend. Fig. 1b shows a matrix where each box represents a possible asset-target pair and the bold frames highlight the optimal solution. The small numbers in the boxes provide the ranking of multiple solutions (actually 3!=6 solutions are possible), i.e., the three boxes labeled "1" are the optimal solution, "2" are the second best solution and "3" are the third best solution. In this way, users (e.g., targeteers in a command center) can see both the <u>results</u> (black bars) and the <u>reasons</u> (colored bars) for each asset-target pair, for both the <u>normative</u> (rank 1) and <u>alternative</u> (rank 2, 3, etc.) solutions, before they make a final decision.

3 Solution Summary

The display design also shows a "Solution Summary" (Fig. 1c) that compares the normative solution (rank 1) to alternative solutions (ranks 2 and 3). In this case, the user can see (Fig. 1c) that three solutions actually have equivalent Pairing Gains (black bars); hence the user may select solution 3 instead of solution 1 if his preference (not captured by the value function) is to minimize Divert Cost (yellow bar). Similarly, even if the user selects solution 1, he may prefer not to divert A (from its original mission to β) since the gain (black bar) for the pair {A, β} in Fig. 1b is so small.

Reference

1. Bertsekas, D. P.: Auction Algorithms for Network Flow Problems: A Tutorial Introduction. Comput. Optim. Appl. 1 (1992) 7-66

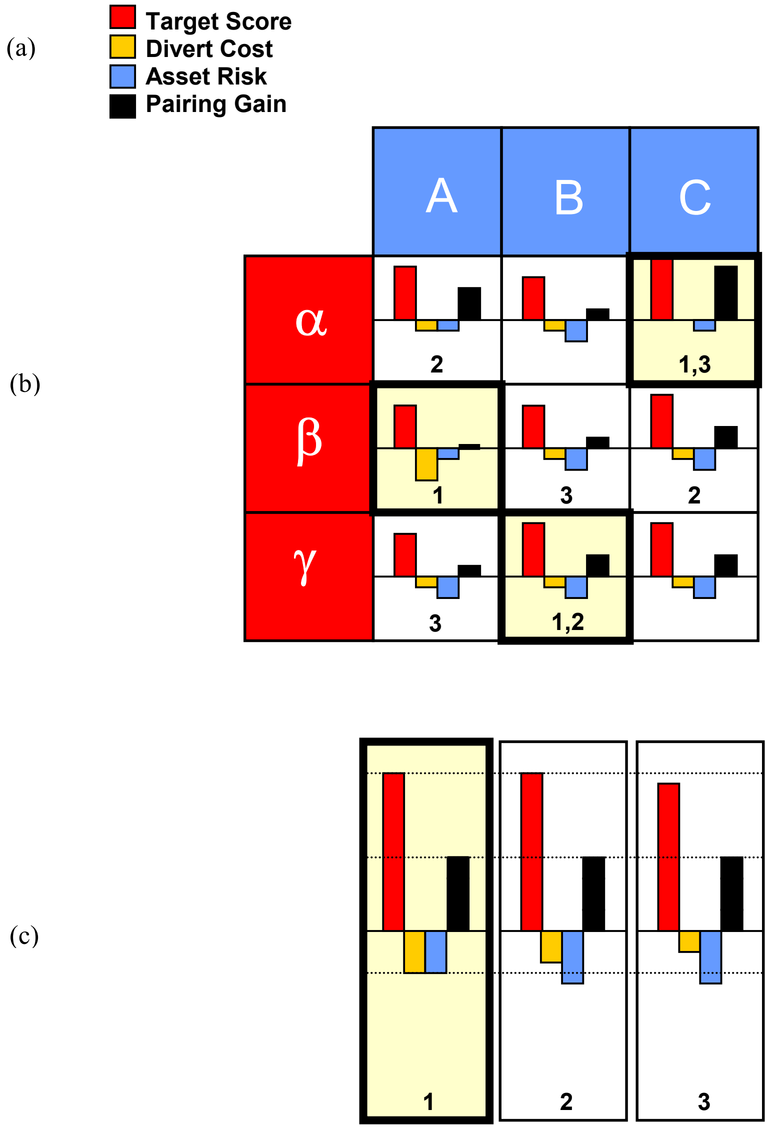

Fig. 1. An informative illustration of the pairing problem. (a) Color coding: Legend shows the terms in the asset-target value function. (b) Bar-Gain Boxes: Matrix of boxes with bars showing gain (black) and other terms (colors) in the value function for each asset-target pair. Assets (columns) are denoted A, B, C. Targets (rows) are denoted α, β, γ. Small numbers at the bottom of boxes denote solution rankings, each solution comprising N boxes in the NxN matrix. (c) Solution Summary: Comparison of possible solutions (ranked 1, 2, 3 by the support system).